## Math 1411

## Practice Exam 1

1) Find the following limits analytically (if they exist).

a) 
$$\lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$$

b) 
$$\lim_{x\to 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x}$$

c) 
$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \cot x}{\cos x - \sin x}$$

d) 
$$\lim_{x \to -1} \frac{x^2 + 6x + 5}{x^2 + 3x + 2}$$

e) 
$$\lim_{x\to 0} \frac{\tan(5x)\cos(5x)}{3x}$$

2) Find the constants *a* and *b* such that is continuous on the entire real line.

$$f(x) = \begin{cases} 2a , x \le 3\\ x - 1, 3 < x < 5\\ -2b , x \ge 5 \end{cases}$$

3) Let 
$$f(x) = \frac{x+4}{x^2+x-12}$$
.

- a) Find any points of discontinuity on the graph of f.
- b) Determine whether these discontinuities are removable or nonremovable. Explain.
- 4) Find the derivatives of the following functions.

a) 
$$f(x) = 6 \sec^2 \left(\frac{5x}{2x+3}\right)$$

b) 
$$g(x) = \left(\frac{3x}{x^2 + 2x}\right)^2$$

c) 
$$h(x) = \sqrt{7 - 5x^2}$$

d) 
$$k(x) = \frac{2}{\sqrt{x^3 + 2x}}$$

e)  $y = (\csc x)\sqrt{x^2 + 2x + 1}$ 

f) 
$$f(x) = 3\sin^2(2x^2 + 3)$$

g) 
$$y = \frac{2(1+\sin x)}{5 \cos x}$$

- 5) A ladder 6 feet long is resting against a wall. If the top of the ladder is sliding down the wall at a rate of 3 feet per second, how fast is the bottom of the ladder moving away from the wall when the bottom of the ladder is 2 feet from the wall?
- 6) A stone is thrown into a pond, and circular ripples expand away from the point of impact. The radius of one of the circles increases at a rate of 2 feet per second. How fast is the area of this circle increasing when the radius is 4 feet? (The area of a circle is  $A = \pi r^2$ )
- 7) At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic meters per minute. The diameter of the base of the cone is approximately three times the height. At what rate is the height of the pile changing when the pile is 15 meters high? The formula for the volume of a cone is  $V = \frac{1}{3}\pi r^2 h$ .
- 8) The spherical balloon is inflated with gas at a rate of 600 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is 20 centimeters?
- 9) An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna (see figure). When the plane is 10 miles away (s = 10), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the plane?



10) Find dy/dx by implicit differentiation:  $3x \sin y = y \cos x$ .

11) Find the equation of the line that is tangent to the curve  $yx^3 + xy^2 = 2y^2$  at the point (1, 2).